




*Spline, N.U.R.B.S, etc.*

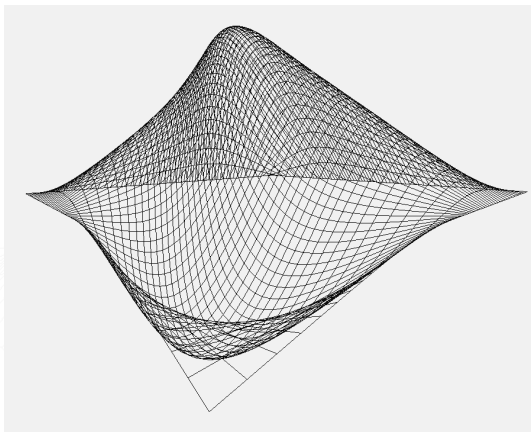
David Odin

Forma3Dev pour CPE-Lyon

CC BY-NC-ND — 2018

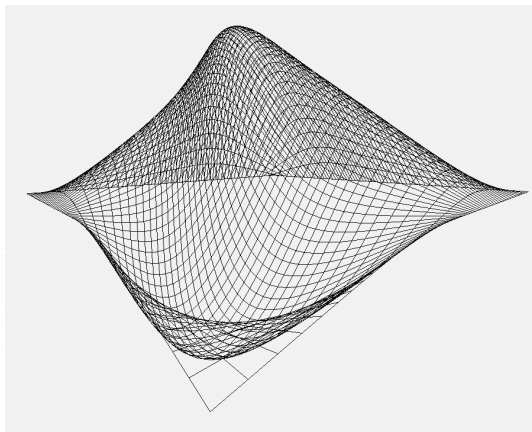
# SPLINE, N.U.R.B.S, ETC. – PLAN

- 1 PRÉSENTATION
  - 2 COURBES CONNUES
  - 3 B-SPLINE
  - 4 SURFACES
  - 5 ANNEXES
- 

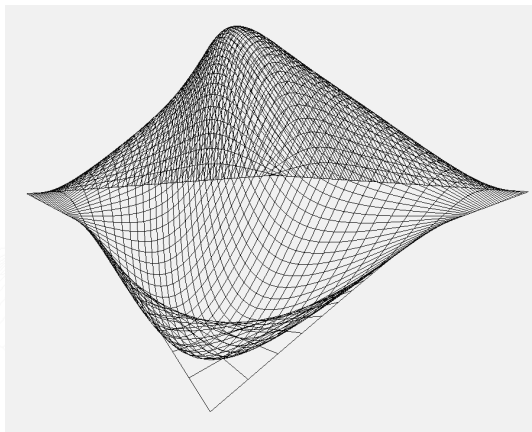


- Surface lisse

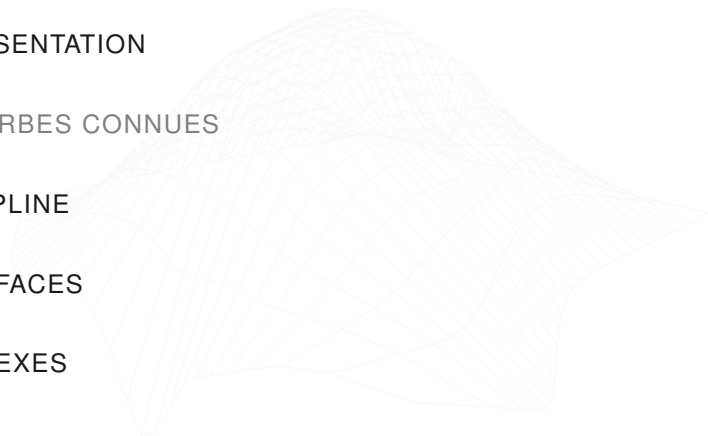
# OBJECTIF



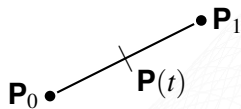
- Surface lisse
- Controlable localement



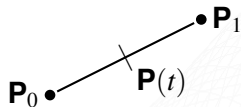
- Surface lisse
- Controlable localement
- Forme attendue (prévisible)

- 1 PRÉSENTATION
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# SEGMENTS



# SEGMENTS

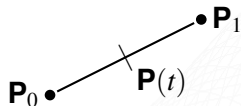


$$\begin{cases} \mathbf{P}(0) = \mathbf{P}_0 \\ \mathbf{P}(1) = \mathbf{P}_1 \end{cases}$$

$$\mathbf{P}(t) = (1 - t) \cdot \mathbf{P}_0 + t \cdot \mathbf{P}_1$$



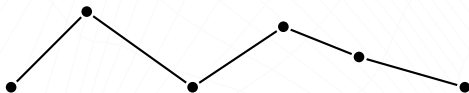
# SEGMENTS



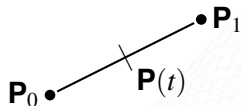
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Extension : Courbe brisée



# SEGMENTS



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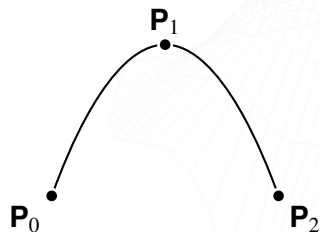
Extension : Courbe brisée



Problème : courbe non lisse

# PARABOLE

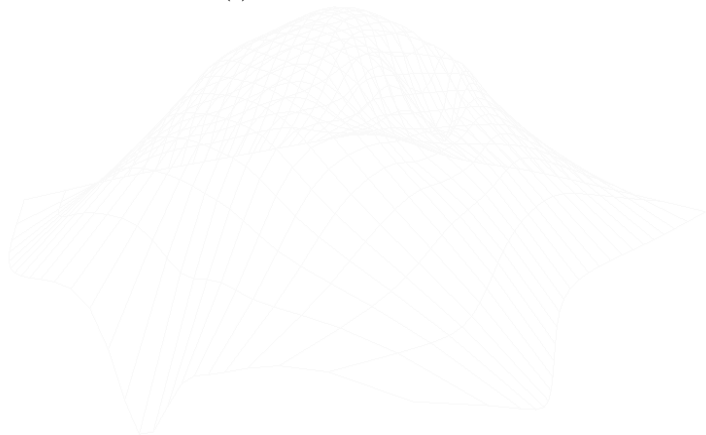
degré 2  $\Rightarrow$  3 points



$$\begin{cases} \mathbf{P}(0) = \mathbf{P}_0 \\ \mathbf{P}(\frac{1}{2}) = \mathbf{P}_1 \\ \mathbf{P}(1) = \mathbf{P}_2 \end{cases}$$

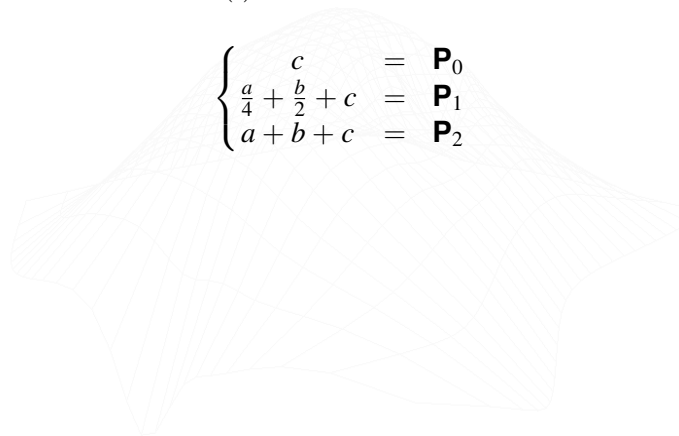
# PARABOLE RÉOLUTION

$$\mathbf{P}(t) = a \cdot t^2 + b \cdot t + c$$



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$$\mathbf{P}(t) = a \cdot t^2 + b \cdot t + c$$

$$\begin{cases} c & = \mathbf{P}_0 \\ \frac{a}{4} + \frac{b}{2} + c & = \mathbf{P}_1 \\ a + b + c & = \mathbf{P}_2 \end{cases}$$


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$$\begin{cases} c & = \mathbf{P}_0 \\ a + 2 \cdot b & = 4 \cdot (\mathbf{P}_1 - \mathbf{P}_0) \\ a + b & = \mathbf{P}_2 - \mathbf{P}_0 \end{cases}$$

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$$b = 4 \cdot \mathbf{P}_1 - 4 \cdot \mathbf{P}_0 - \mathbf{P}_2 + \mathbf{P}_0 = 4 \cdot \mathbf{P}_1 - 3 \cdot \mathbf{P}_0 - \mathbf{P}_2$$

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$$a = \mathbf{P}_2 - \mathbf{P}_0 - 4 \cdot \mathbf{P}_1 + 3 \cdot \mathbf{P}_0 + \mathbf{P}_2 = 2 \cdot \mathbf{P}_2 + 2 \cdot \mathbf{P}_0 - 4 \cdot \mathbf{P}_1$$



# PARABOLE RÉOLUTION

$$\mathbf{P}(t) = a \cdot t^2 + b \cdot t + c$$

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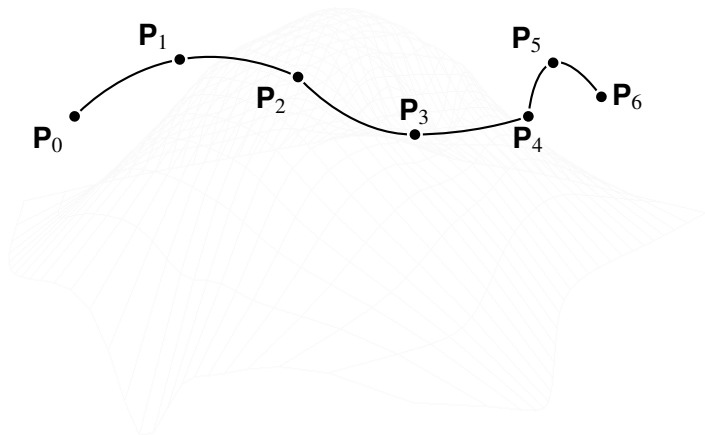
$$b = 4 \cdot \mathbf{P}_1 - 4 \cdot \mathbf{P}_0 - \mathbf{P}_2 + \mathbf{P}_0 = 4 \cdot \mathbf{P}_1 - 3 \cdot \mathbf{P}_0 - \mathbf{P}_2$$

$$a = \mathbf{P}_2 - \mathbf{P}_0 - 4 \cdot \mathbf{P}_1 + 3 \cdot \mathbf{P}_0 + \mathbf{P}_2 = 2 \cdot \mathbf{P}_2 + 2 \cdot \mathbf{P}_0 - 4 \cdot \mathbf{P}_1$$

$$[\mathbf{P}(t) = 2 \cdot (\mathbf{P}_0 - 2 \cdot \mathbf{P}_1 + \mathbf{P}_2) \cdot t^2 + (-3 \cdot \mathbf{P}_0 + 4 \cdot \mathbf{P}_1 - \mathbf{P}_2) \cdot t + \mathbf{P}_0]$$

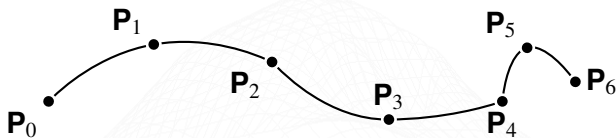
# COURBES RACCORDABLES

Problème

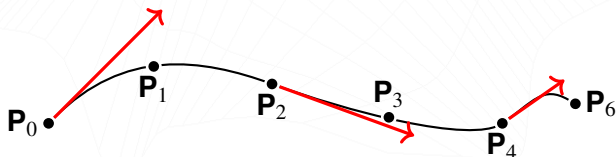


# COURBES RACCORDABLES

Problème



On aimerait



2 positions + 2 dérivées (directions)  $\Rightarrow$  polynôme de degré 3

# CUBIQUE D'HERMITE

$$\begin{cases} \mathbf{P}(0) &= \mathbf{P}_0 \\ \mathbf{P}(1) &= \mathbf{P}_1 \\ \mathbf{P}'(0) &= d_0 \\ \mathbf{P}'(1) &= d_1 \end{cases}$$

$$\mathbf{P}(t) = a \cdot t^3 + b \cdot t^2 + c \cdot t + d$$

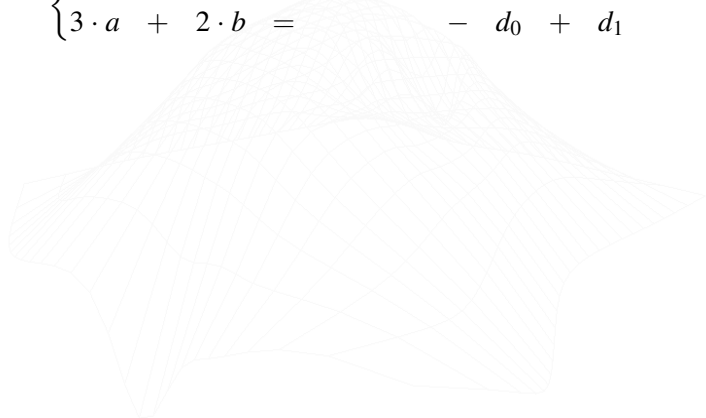
$$\mathbf{P}'(t) = 3 \cdot a \cdot t^2 + 2 \cdot b \cdot t + c$$

soit

$$\begin{cases} a + b + c + d = \mathbf{P}_1 \\ 3 \cdot a + 2 \cdot b + c = d_1 \\ c = d_0 \\ d = \mathbf{P}_0 \end{cases}$$

# COURBES RACCORDABLES, DÉMONSTRATION

$$\begin{cases} a + b = \mathbf{P}_1 - \mathbf{P}_0 - d_0 \\ 3 \cdot a + 2 \cdot b = \phantom{\mathbf{P}_1 - \mathbf{P}_0} - d_0 + d_1 \end{cases}$$



# COURBES RACCORDABLES, DÉMONSTRATION

$$\begin{cases} a + b = \mathbf{P}_1 - \mathbf{P}_0 - d_0 \\ 3 \cdot a + 2 \cdot b = \phantom{\mathbf{P}_1 - \mathbf{P}_0 - d_0} - d_0 + d_1 \end{cases}$$

$$a = -2 \cdot \mathbf{P}_1 + 2 \cdot \mathbf{P}_0 + d_0 + d_1 = 2 \cdot (\mathbf{P}_0 - \mathbf{P}_1) + d_1 + d_0$$

$$b = \mathbf{P}_1 - \mathbf{P}_0 - d_0 + 2 \cdot \mathbf{P}_1 - 2 \cdot \mathbf{P}_0 - d_0 - d_1 = 3 \cdot (\mathbf{P}_1 - \mathbf{P}_0) - 2 \cdot d_0 - d_1$$

# COURBES RACCORDABLES, DÉMONSTRATION

$$\begin{cases} a + b = \mathbf{P}_1 - \mathbf{P}_0 - d_0 \\ 3 \cdot a + 2 \cdot b = -d_0 + d_1 \end{cases}$$

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$$\left[ \begin{array}{l} \mathbf{P}(t) = [2 \cdot (\mathbf{P}_0 - \mathbf{P}_1) + d_0 + d_1] \cdot t^3 + \\ [3 \cdot (\mathbf{P}_1 - \mathbf{P}_0) - 2 \cdot d_0 - d_1] \cdot t^2 + d_0 \cdot t + \mathbf{P}_0 \end{array} \right]$$

# COURBES RACCORDABLES, DÉMONSTRATION

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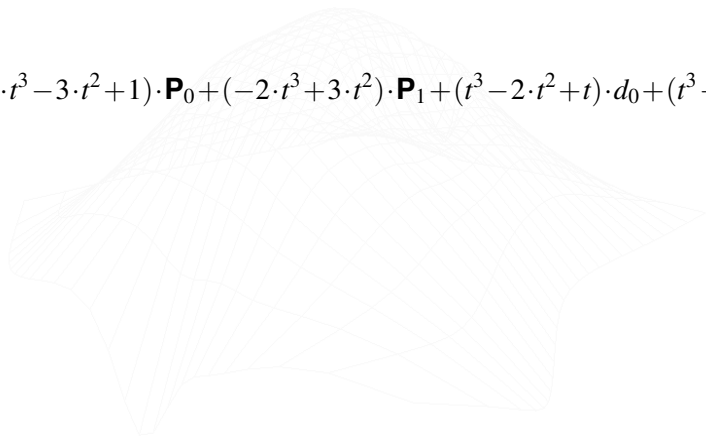
$$b = \mathbf{P}_1 - \mathbf{P}_0 - d_0 + 2 \cdot \mathbf{P}_1 - 2 \cdot \mathbf{P}_0 - d_0 - d_1 = 3 \cdot (\mathbf{P}_1 - \mathbf{P}_0) - 2 \cdot d_0 - d_1$$

$$\left[ \begin{array}{l} \mathbf{P}(t) = [2 \cdot (\mathbf{P}_0 - \mathbf{P}_1) + d_0 + d_1] \cdot t^3 + \\ [3 \cdot (\mathbf{P}_1 - \mathbf{P}_0) - 2 \cdot d_0 - d_1] \cdot t^2 + d_0 \cdot t + \mathbf{P}_0 \end{array} \right]$$

**Note :** Manipuler des dérivées, c'est pratique pour des courbes, mais nettement moins pour des surfaces !



$$\mathbf{P}(t) = (2 \cdot t^3 - 3 \cdot t^2 + 1) \cdot \mathbf{P}_0 + (-2 \cdot t^3 + 3 \cdot t^2) \cdot \mathbf{P}_1 + (t^3 - 2 \cdot t^2 + t) \cdot d_0 + (t^3 - t^2) \cdot d_1$$



# REPRÉSENTATION DE BÉZIER

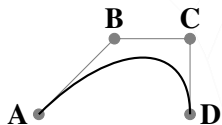
$$\mathbf{P}(t) = (2 \cdot t^3 - 3 \cdot t^2 + 1) \cdot \mathbf{P}_0 + (-2 \cdot t^3 + 3 \cdot t^2) \cdot \mathbf{P}_1 + (t^3 - 2 \cdot t^2 + t) \cdot d_0 + (t^3 - t^2) \cdot d_1$$

$$\mathbf{P}(t) = (1-t)^3 \underbrace{\mathbf{P}_0}_{\text{A}} + 3 \cdot (1-t)^2 \cdot t \cdot \underbrace{\left(\mathbf{P}_0 + \frac{d_0}{3}\right)}_{\text{B}} + 3 \cdot (1-t) \cdot t^2 \cdot \underbrace{\left(\mathbf{P}_1 - \frac{d_1}{3}\right)}_{\text{C}} + t^3 \cdot \underbrace{\mathbf{P}_1}_{\text{D}}$$

# REPRÉSENTATION DE BÉZIER

$$\mathbf{P}(t) = (2 \cdot t^3 - 3 \cdot t^2 + 1) \cdot \mathbf{P}_0 + (-2 \cdot t^3 + 3 \cdot t^2) \cdot \mathbf{P}_1 + (t^3 - 2 \cdot t^2 + t) \cdot d_0 + (t^3 - t^2) \cdot d_1$$

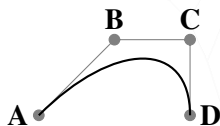
$$\mathbf{P}(t) = (1-t)^3 \underbrace{\mathbf{P}_0}_{\text{A}} + 3 \cdot (1-t)^2 \cdot t \cdot \underbrace{\left(\mathbf{P}_0 + \frac{d_0}{3}\right)}_{\text{B}} + 3 \cdot (1-t) \cdot t^2 \cdot \underbrace{\left(\mathbf{P}_1 - \frac{d_1}{3}\right)}_{\text{C}} + t^3 \cdot \underbrace{\mathbf{P}_1}_{\text{D}}$$



# REPRÉSENTATION DE BÉZIER

$$\mathbf{P}(t) = (2 \cdot t^3 - 3 \cdot t^2 + 1) \cdot \mathbf{P}_0 + (-2 \cdot t^3 + 3 \cdot t^2) \cdot \mathbf{P}_1 + (t^3 - 2 \cdot t^2 + t) \cdot d_0 + (t^3 - t^2) \cdot d_1$$

$$\mathbf{P}(t) = (1-t)^3 \underbrace{\mathbf{P}_0}_A + 3 \cdot (1-t)^2 \cdot t \cdot \underbrace{\left(\mathbf{P}_0 + \frac{d_0}{3}\right)}_B + 3 \cdot (1-t) \cdot t^2 \cdot \underbrace{\left(\mathbf{P}_1 - \frac{d_1}{3}\right)}_C + t^3 \cdot \underbrace{\mathbf{P}_1}_D$$

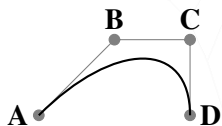


● Polygone de controle **A B C D**

# REPRÉSENTATION DE BÉZIER

$$\mathbf{P}(t) = (2 \cdot t^3 - 3 \cdot t^2 + 1) \cdot \mathbf{P}_0 + (-2 \cdot t^3 + 3 \cdot t^2) \cdot \mathbf{P}_1 + (t^3 - 2 \cdot t^2 + t) \cdot d_0 + (t^3 - t^2) \cdot d_1$$

$$\mathbf{P}(t) = (1-t)^3 \underbrace{\mathbf{P}_0}_A + 3 \cdot (1-t)^2 \cdot t \cdot \underbrace{\left(\mathbf{P}_0 + \frac{d_0}{3}\right)}_B + 3 \cdot (1-t) \cdot t^2 \cdot \underbrace{\left(\mathbf{P}_1 - \frac{d_1}{3}\right)}_C + t^3 \cdot \underbrace{\mathbf{P}_1}_D$$

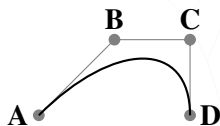


- Polygone de controle **A B C D**
- Courbe tangente à **[A B]** et à **[C D]**

# REPRÉSENTATION DE BÉZIER

$$\mathbf{P}(t) = (2 \cdot t^3 - 3 \cdot t^2 + 1) \cdot \mathbf{P}_0 + (-2 \cdot t^3 + 3 \cdot t^2) \cdot \mathbf{P}_1 + (t^3 - 2 \cdot t^2 + t) \cdot d_0 + (t^3 - t^2) \cdot d_1$$

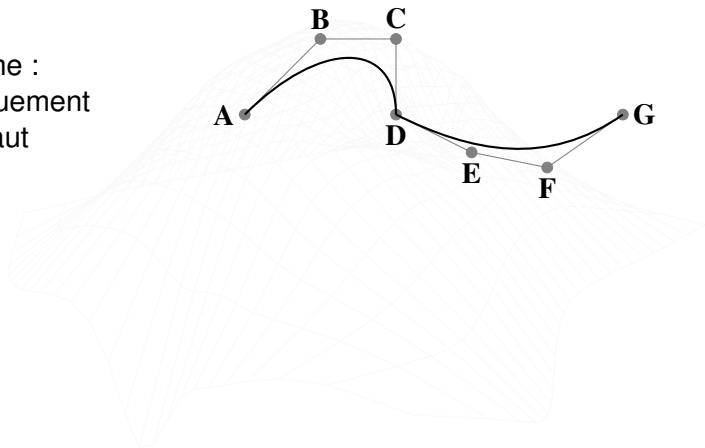
$$\mathbf{P}(t) = (1-t)^3 \underbrace{\mathbf{P}_0}_A + 3 \cdot (1-t)^2 \cdot t \cdot \underbrace{\left(\mathbf{P}_0 + \frac{d_0}{3}\right)}_B + 3 \cdot (1-t) \cdot t^2 \cdot \underbrace{\left(\mathbf{P}_1 - \frac{d_1}{3}\right)}_C + t^3 \cdot \underbrace{\mathbf{P}_1}_D$$



- Polygone de controle **A B C D**
- Courbe tangente à **[A B]** et à **[C D]**
- Courbe qui passe par **A** et **D**

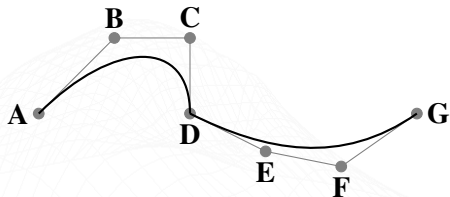
# RACCORDEMENT DE COURBES DE BÉZIER

Problème :  
 $C^0$  uniquement  
par défaut



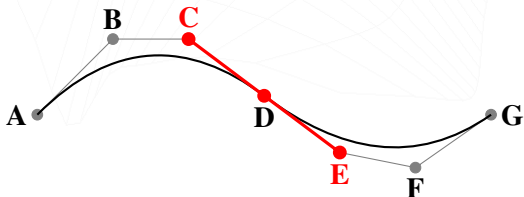
# RACCORDEMENT DE COURBES DE BÉZIER

Problème :  
 $C^0$  uniquement  
par défaut



Solution

Pour du  $C^1$  : E symétrique de C par rapport à D

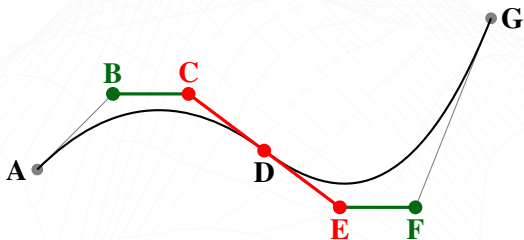




# RACCORDEMENT $C^2$ DE COURBES DE BÉZIER

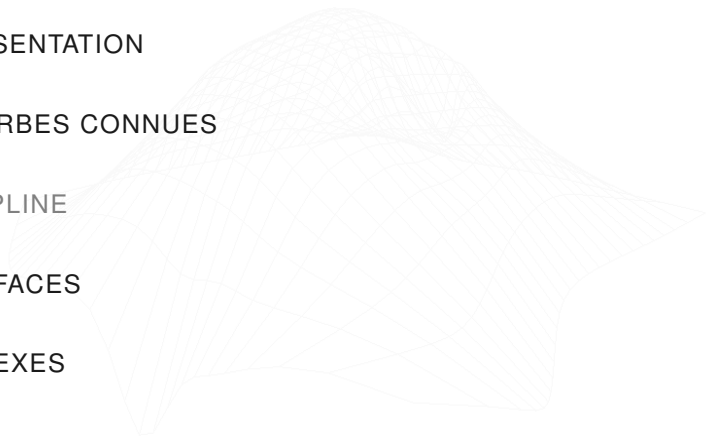
Pour du  $C^2$  :

- **E** symétrique de **C** par rapport à **D**
- **F** symétrique de **B** par rapport à **D**



On est donc assez restreint sur les positions des points.

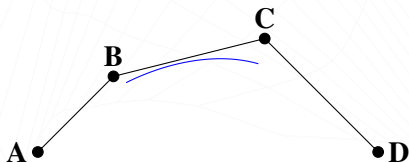
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# B-SPLINE, DÉFINITION

$$\begin{aligned} \mathbf{P}(t) = & \frac{1}{6}(1 - 3t + 3t^2 - t^3) \cdot \mathbf{A} \\ & + \frac{1}{6}(4 - 6t^2 + 3t^3) \cdot \mathbf{B} \\ & + \frac{1}{6}(1 + 3t + 3t^2 - 3t^3) \cdot \mathbf{C} \\ & + \frac{1}{6}t^3 \cdot \mathbf{D} \end{aligned}$$

$$\mathbf{P}(t) = \sum_{k=0}^n b_k(t) \cdot \mathbf{P}_k$$

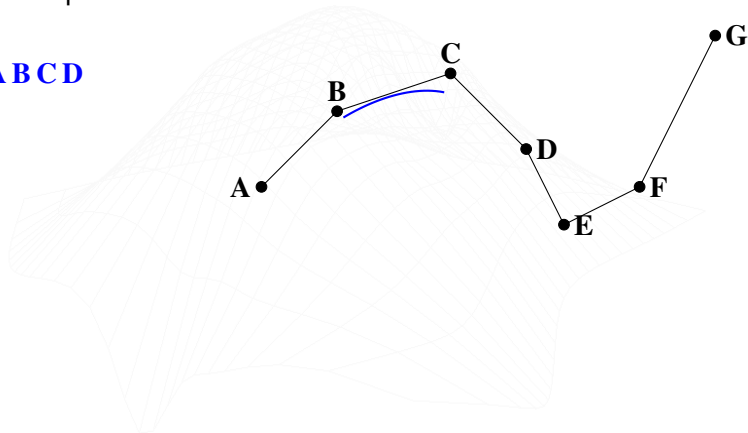


**Note :** La courbe ne commence pas par A ni ne finit par D.

# B-SPLINE, AVANTAGES

- Automatiquement raccordable  $C^2$

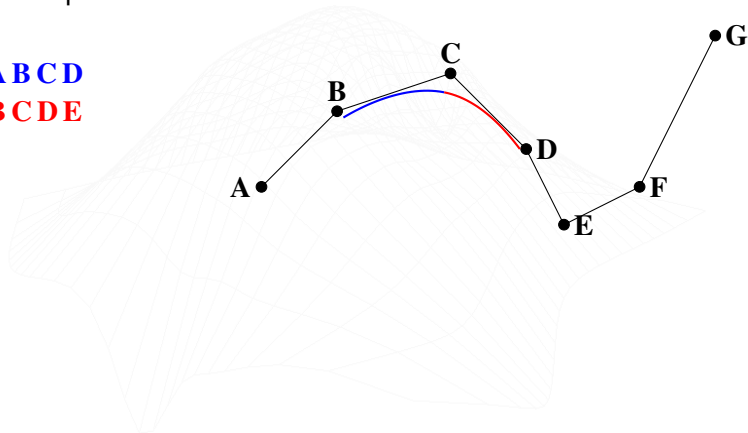
**ABCD**



# B-SPLINE, AVANTAGES

- Automatiquement raccordable  $C^2$

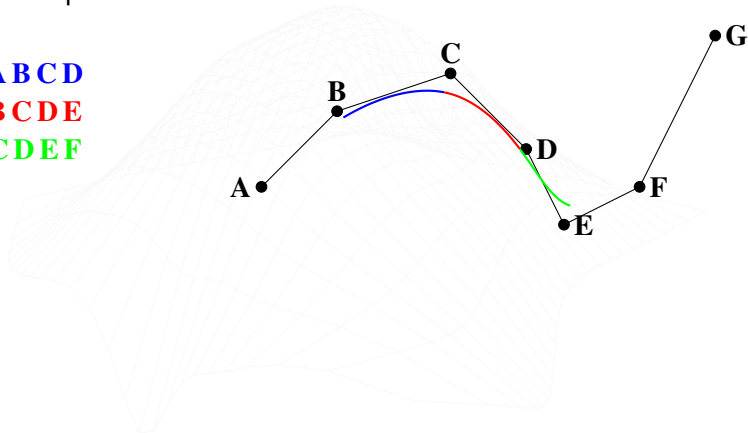
**ABCD**  
**BCDE**



# B-SPLINE, AVANTAGES

- Automatiquement raccordable  $C^2$

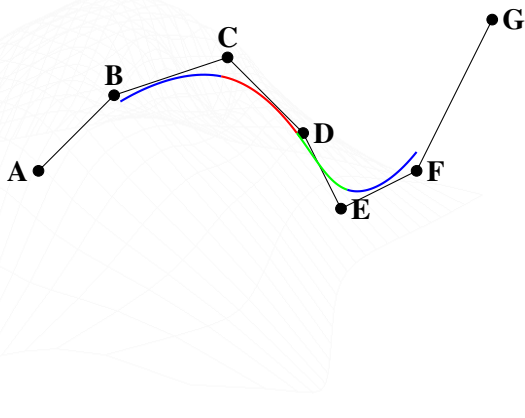
**ABCD**  
**BCDE**  
**CDEF**



# B-SPLINE, AVANTAGES

- Automatiquement raccordable  $C^2$

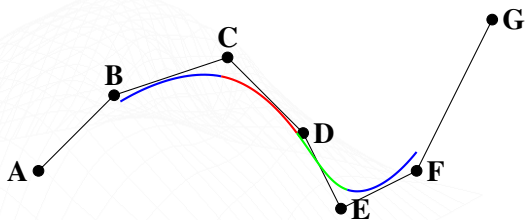
**ABCD**  
**BCDE**  
**CDEF**  
**DEFG**



# B-SPLINE, AVANTAGES

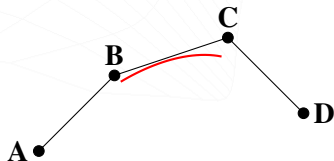
- Automatiquement raccordable  $C^2$

**ABCD**  
**BCDE**  
**CDEF**  
**DEFG**



- Peut passer par des points (en les multipliant)

**ABCD**

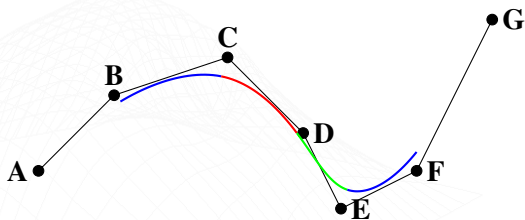




# B-SPLINE, AVANTAGES

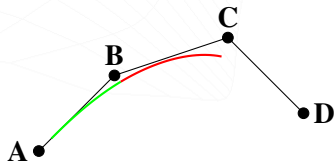
- Automatiquement raccordable  $C^2$

**ABCD**  
**BCDE**  
**CDEF**  
**DEFG**



- Peut passer par des points (en les multipliant)

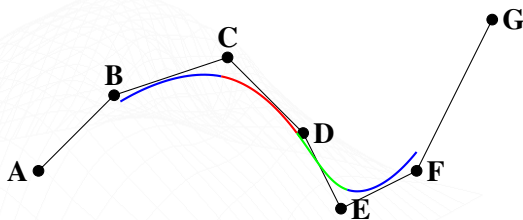
**ABCD**  
**AABC**



# B-SPLINE, AVANTAGES

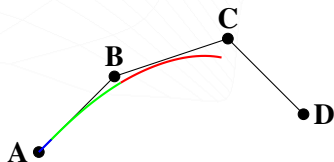
- Automatiquement raccordable  $C^2$

**ABCD**  
**BCDE**  
**CDEF**  
**DEFG**



- Peut passer par des points (en les multipliant)

**ABCD**  
**AABC**  
**AAAB**



# EXPRESSION MATRICIELLE

Bézier :

$$\mathbf{P}(t) = (1-t)^3 \cdot \mathbf{P}_0 + 3 \cdot (1-t)^2 \cdot t \mathbf{P}_1 + 3 \cdot (1-t) \cdot t^2 \mathbf{P}_2 + t^3 \mathbf{P}_3$$

$$\begin{aligned} &= \begin{pmatrix} 1 & -3 \cdot t & +3 \cdot t^2 & -t^3 \end{pmatrix} \mathbf{P}_0 \\ &+ 3 \cdot \begin{pmatrix} & t & -2 \cdot t^2 & +t^3 \end{pmatrix} \mathbf{P}_1 \\ &+ 3 \cdot \begin{pmatrix} & & t^2 & -t^3 \end{pmatrix} \mathbf{P}_2 \\ &+ \begin{pmatrix} & & & t^3 \end{pmatrix} \mathbf{P}_3 \end{aligned}$$

$$\begin{aligned} &= \begin{pmatrix} -\mathbf{P}_0 + 3 \cdot \mathbf{P}_1 - 3 \cdot \mathbf{P}_2 + \mathbf{P}_3 \\ 3 \cdot (\mathbf{P}_0 - 2 \cdot \mathbf{P}_1 + \mathbf{P}_2) \\ 3 \cdot (-\mathbf{P}_0 + \mathbf{P}_1) \\ \mathbf{P}_0 \end{pmatrix} \cdot \begin{matrix} t^3 + \\ t^2 + \\ t + \\ \end{matrix} \end{aligned}$$

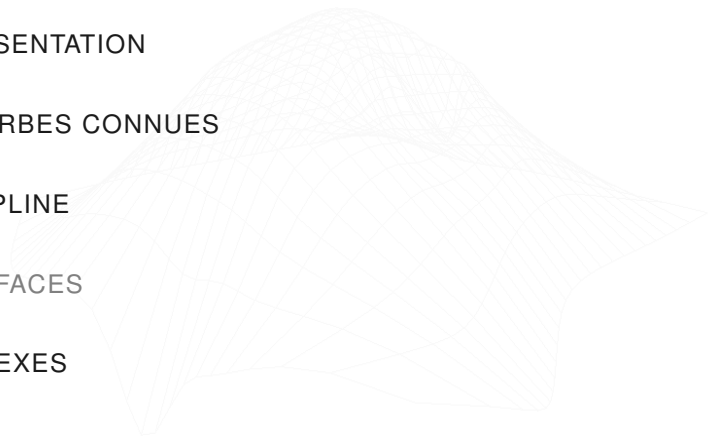
$$= (\mathbf{P}_0 \quad \mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3) \cdot \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} t^3 \\ t^2 \\ t \\ 1 \end{pmatrix}$$

$$\begin{aligned} &= \mathbf{P}^T \cdot M \cdot T \\ &= T^T \cdot M \cdot \mathbf{P} = \begin{pmatrix} t^3 & t^2 & t & 1 \end{pmatrix} \cdot (M) \cdot \begin{pmatrix} \mathbf{P}_0 \\ \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{pmatrix} \end{aligned}$$

B-Spline

On remplace simplement  $M$  par :

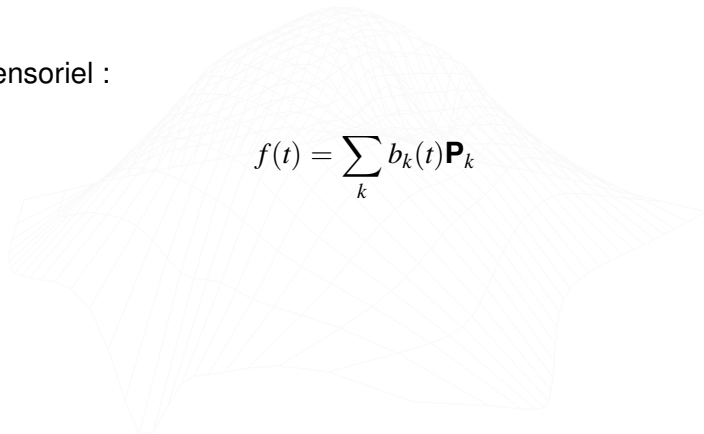
$$M_s = \frac{1}{6} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 0 & 4 \\ -3 & 3 & 3 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- 1 PRÉSENTATION
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- 

Produit tensoriel :

● 1D :

$$f(t) = \sum_k b_k(t) \mathbf{P}_k$$



Produit tensoriel :

● 1D :

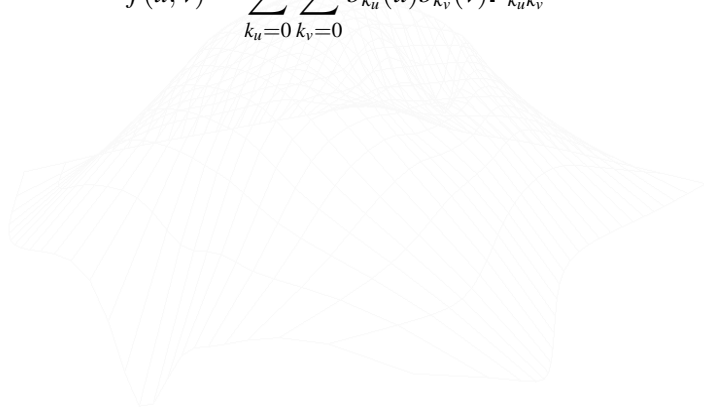
$$f(t) = \sum_k b_k(t) \mathbf{P}_k$$

● 2D :

$$f(u, v) = \sum_{k_u} \sum_{k_v} b_{k_u}(u) b_{k_v}(v) \mathbf{P}_{k_u k_v}$$

# POUR UNE SURFACE DE BÉZIER

$$f(u, v) = \sum_{k_u=0}^3 \sum_{k_v=0}^3 b_{k_u}(u) b_{k_v}(v) \mathbf{P}_{k_u k_v}$$





# POUR UNE SURFACE DE BÉZIER

$$f(u, v) = \sum_{k_u=0}^3 \sum_{k_v=0}^3 b_{k_u}(u) b_{k_v}(v) \mathbf{P}_{k_u k_v}$$

$$f(u, v) = \sum_{k_u=0}^3 b_{k_u}(u) \underbrace{\left( \sum_{k_v=0}^3 b_{k_v}(v) \mathbf{P}_{k_u k_v} \right)}_{[\mathbf{P}_{k_u 0} \dots \mathbf{P}_{k_u 3}] \cdot \mathbf{M} \cdot \mathbf{V}}$$

# POUR UNE SURFACE DE BÉZIER

$$f(u, v) = \sum_{k_u=0}^3 \sum_{k_v=0}^3 b_{k_u}(u) b_{k_v}(v) \mathbf{P}_{k_u k_v}$$

$$f(u, v) = \sum_{k_u=0}^3 b_{k_u}(u) \underbrace{\left( \sum_{k_v=0}^3 b_{k_v}(v) \mathbf{P}_{k_u k_v} \right)}_{[\mathbf{P}_{k_u 0} \dots \mathbf{P}_{k_u 3}] \cdot \mathbf{M} \cdot \mathbf{V}}$$

$$f(u, v) = \mathbf{U}^T \cdot \mathbf{M} \cdot [\mathbf{P}] \cdot \mathbf{M} \cdot \mathbf{V}$$

# POUR UNE SURFACE DE BÉZIER

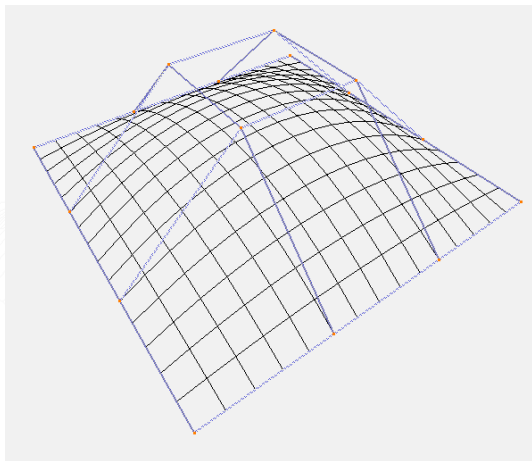
$$f(u, v) = \sum_{k_u=0}^3 \sum_{k_v=0}^3 b_{k_u}(u) b_{k_v}(v) \mathbf{P}_{k_u k_v}$$

$$f(u, v) = \sum_{k_u=0}^3 b_{k_u}(u) \underbrace{\left( \sum_{k_v=0}^3 b_{k_v}(v) \mathbf{P}_{k_u k_v} \right)}_{[\mathbf{P}_{k_u 0} \dots \mathbf{P}_{k_u 3}] \cdot \mathbf{M} \cdot \mathbf{V}}$$

$$f(u, v) = \mathbf{U}^T \cdot \mathbf{M} \cdot [\mathbf{P}] \cdot \mathbf{M} \cdot \mathbf{V}$$

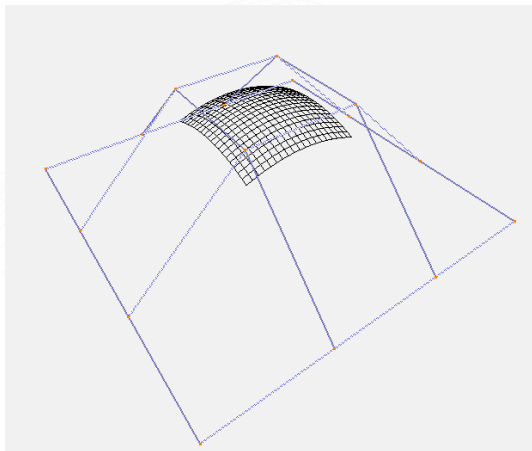
$$f(u, v) = \begin{pmatrix} u^3 & u^2 & u & 1 \end{pmatrix} \cdot \mathbf{M} \begin{pmatrix} \mathbf{P}_{00} & \mathbf{P}_{01} & \mathbf{P}_{02} & \mathbf{P}_{03} \\ \mathbf{P}_{10} & \mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{P}_{13} \\ \mathbf{P}_{20} & \mathbf{P}_{21} & \mathbf{P}_{22} & \mathbf{P}_{23} \\ \mathbf{P}_{30} & \mathbf{P}_{31} & \mathbf{P}_{32} & \mathbf{P}_{33} \end{pmatrix} \cdot \mathbf{M} \begin{pmatrix} v^3 \\ v^2 \\ v \\ 1 \end{pmatrix}$$

# SURFACE DE BÉZIER, REPRÉSENTATION GRAPHIQUE

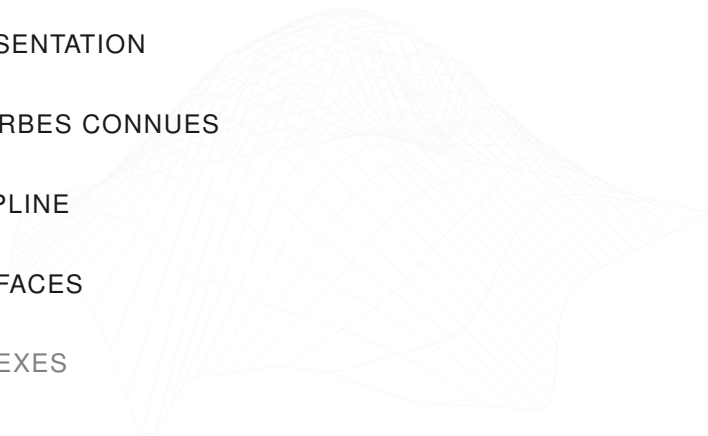


# SURFACE SPLINE, RACCORDABLE

Comme pour les surfaces de Bézier, seule la matrice  $M$  change.



# SPLINE, N.U.R.B.S, ETC. – PLAN

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- 

- *The Nurbs Book* Les Piegl & Wayne Tiller

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- « Cours écrit » de Damien Rohmer